# EXPERIMENT 3

# Incline

**GOAL**

- To study the acceleration of an object sliding down an incline
- To determine the coefficient of the kinetic friction between the inclined surface and the sliding object

**THEORY**

Consider a sled sliding down an inclined surface with an angle of inclination $\theta$ (Fig. 1). While the boy the sled is moving down, three forces are acting on the system sled-boy: the gravity force $\mathbf{F_G}$, the normal force $\mathbf{F_N}$ and the friction force $\mathbf{F_{fr}}$.

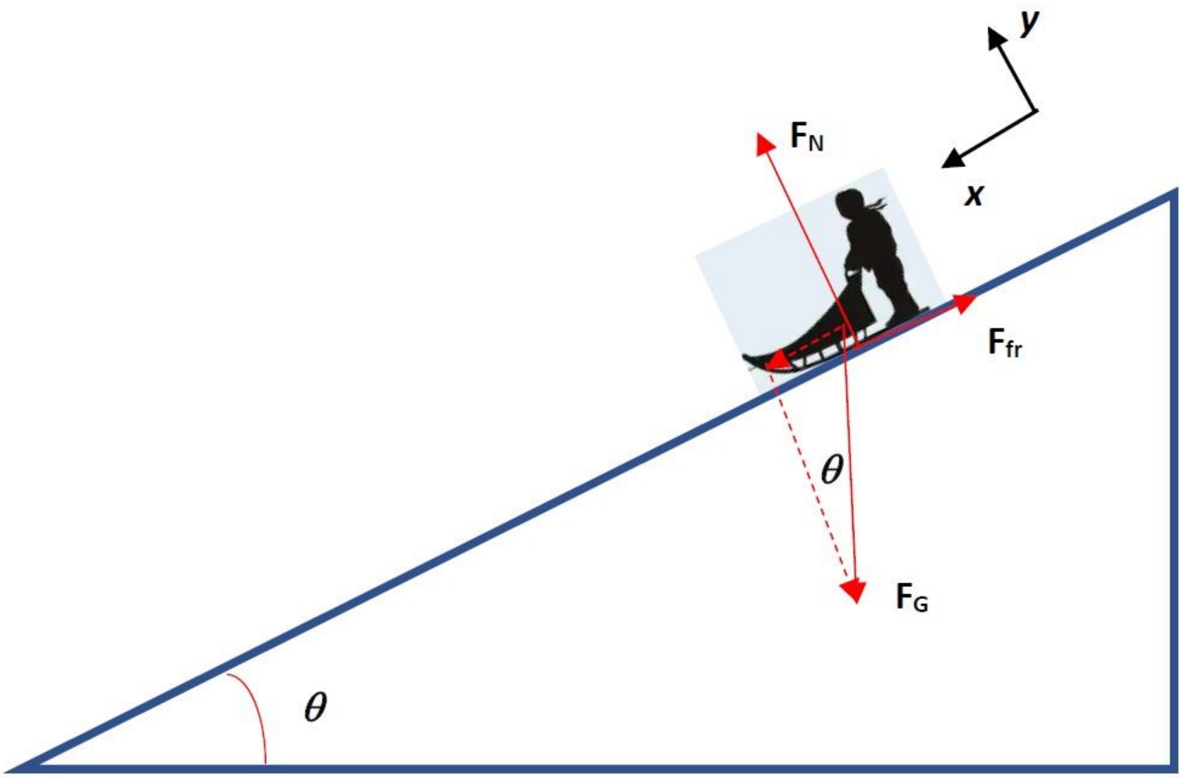


Fig. 1. Motion on the inclined surface

Because the system is moving, we have to consider friction as kinetic. We can apply Newton's First Law to determine the net force:

$$\mathbf{F}_{net} = \mathbf{F}_G + \mathbf{F}_N + \mathbf{F}_{fr} \qquad (1)$$

According to Newton's Second Law,

$$\mathbf{F}_{net} = m\boldsymbol{a} \qquad (2)$$

where $\boldsymbol{a}$ is acceleration and $m$ is the mass of the system (sled and boy).
Let's expand the equation (2) on $x$ and $y$ directions referring to the diagram in Fig. 1:

$$mg\sin\theta - F_{fr} = ma \qquad (3)$$
$$-mg\cos\theta + F_N = 0 \qquad (4)$$

Consider

$$F_{fr} = \mu_k F_N \qquad (5)$$

where $\mu_k$ is the coefficient of the kinetic friction between the surface and sled

Obtaining $F_N = mg\cos\theta$ from (4), we can modify (3) as:

$$mg\sin\theta - \mu_k mg\cos\theta = ma \qquad (6)$$

Now we can solve (6) for the coefficient of the kinetic friction:

$$\mu_k = \frac{g\sin\theta - a}{g\cos\theta} \qquad (7)$$

**PROCEDURE**

**Inclination**
Sometimes it is complicated to measure the angle of inclination directly. As you can see from formula (7), in fact, we need the values of $\sin\theta$ and $\cos\theta$ to determine the coefficient of kinetic friction.

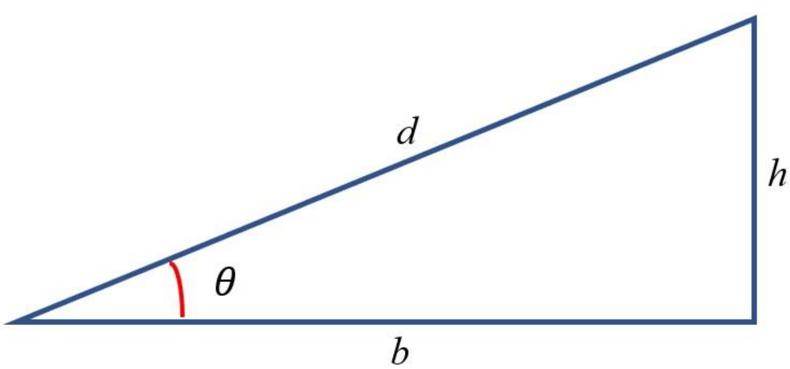

Fig. 2. Geometry of the inclined surface

Referring to Fig. 2, we can obtain:

$$\sin\theta = \frac{h}{d} \tag{8}$$

$$\cos\theta = \frac{b}{d} \tag{9}$$

During the experiment, it is easier to measure the height and the base of the triangle. The hypotenuse d can be evaluated using the Pythagoras Theorem:

$$d^2 = b^2 + h^2 \tag{10}$$

or

$$b = \sqrt{d^2 - h^2} \tag{11}$$

Using (11) in (9), we obtain the following:

$$\cos\theta = \frac{\sqrt{d^2 - h^2}}{d} \tag{12}$$

**Acceleration**

The distance covered by an object moving at the acceleration $a$ during time $t$ can be determined as:

$$d = v_0 t + \frac{1}{2} a t^2 \tag{13}$$

where $v_0$ is the initial velocity of the object. By setting up $v_0 = 0$, we can obtain:

$$a = \frac{2d}{t^2} \tag{14}$$

**Measurements**

1. Construct an inclined surface using a board, as shown in Fig. 2.
2. Measure the height and base of the right triangle, whose hypotenuse is formed by the board.

3. Release a rectangular block or bar from the apex (top vertex) of the triangle, starting simultaneously with a stopwatch. Stop the stopwatch when the block reaches another end of the hypotenuse.
4. Repeat step 3 for five different heights and complete the following table considering $g$ = 9.8 m/s$^2$:

Table 1

| $b$ (m) | $h$ (m) | $t$ (s) | $a$ (m/s$^2$) | $\mu_k$ |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
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5. Evaluate the average value of the coefficient of the kinetic friction using the last column of Table 1.